Problem1:

1. There are eight functions:

f1: a🡪0, b🡪0, c🡪0

f2: a🡪0, b🡪0, c🡪1

f3: a🡪0, b🡪1, c🡪0

f4: a🡪0, b🡪1, c🡪1

f5: a🡪1, b🡪0, c🡪0

f6: a🡪1, b🡪0, c🡪1

f7: a🡪1, b🡪1, c🡪0

f8: a🡪1, b🡪1, c🡪1

1. The cardinality of the function from {a, b, c}🡪{0,1} is equal to the Pow {a, b, c}. For every function of {a, b, c}🡪{0,1}, Pow {a, b, c} always has element which is corresponding to that. For example, for function f1, corresponding to Ø; f2, corresponding to {c}; f3,corresponding to {b}, f4, corresponding to {b, c}; f5, corresponding to {a}; f6, corresponding to {a, c}; f7, corresponding to {a, b}; f8, corresponding to {a, b, c}.
2. (i): There are nm functions from A to B, because, for every m element in A, everyone of them has n choice, so, the number is n\*n\*n…, there are nm functions from A to B.

(ii): There are 2mn relations between A and B, the relation is subset of A\*B and is Pow(A\*B), so, there are 2mn relations between A and B.

(iii): There are 2(m2-m)/2+m symmetric relations on A. For symmetric relations, (x, y) ∈ R and (y, x) ∈ R. From the diagonal line, there are (m2-m)/2 relations, then, adding the diagonal line which is (m2-m)/2. The symmetric relations on A is Pow {(m2-m)/2} which is 2(m2-m)/2+m.

Problem 2:

1. Five elements: 0, -1, 1, 2, 3. When x=2, y=-3, S x, y = {2m-3n: m, n∈Z}. So, when m=0, n=0, S=0; when m=1, n=1, S=-1; when m=-1, n=-1, S=1; when m=1, n=0, S=2; when m=0, n=-1, S=3.
2. Five elements:0, 16, 12, 28, 4. When x=12, y=16, S x ,y = {12m+16n: m, n∈Z}. So, when m=0, n=0, S=0; when m=0, n=1, S=16; when m=1, n=0, S=12; when m=1, n=1, S=28; when m=-1, n=1, S=4.
3. For this question, I should prove that m1x + n1y = kd (m1, n1, k∈Z)

Firstly, because d = gcd (x, y)

So x = k1d (k1∈Z) and y = K2d(k2∈Z)

So m1x + n1y = k1d+k2d (m1, n1, k1, k2∈Z)

Then m1x + n1y = (k1m1+k2n1) d (m1, n1, k1, k2∈Z)

So, m1x + n1y = kd (m1, n1, k∈Z) is true and Sx, y ⊆ {n : n∈ Z and d|n }.

1. I should prove n = kz (k∈Z) and kz ∈ Sx, y

Because the z is the smallest positive number in Sx, y

So, z = k1(m1x + n1y) (m1, n1, k1∈Z)

Then, z = (k1m1)x + (k1n1)y (m1, n1, k1∈Z)

Then, n = (k1m1)x + (k1n1)y (m1, n1, k1∈Z) ⊆ Sx, y and {n : n∈ Z and z|n } ⊆ Sx, y.

1. Firstly, z = m1x + n1y (m1, n1∈Z)

Then, z = (k1m1+k2n1)d (m1, n1, k1, k2∈Z)

So, d = z/(k1m1+k2n1) (m1, n1, k1, k2∈Z)

So, d <= z/(k1m1+k2n1) (m1, n1, k1, k2∈Z)

1. Firstly, Sx, y = mx + ny (m, n∈Z)

Because d = gcd (x, y)

So, Sx, y = k1md+k2nd (m, n, k1, k­2∈Z)

Sx, y = (k1m+k2n)d (m, n, k1, k­2∈Z)

When k1m+k2n=1, Sx, y = d (m, n, k1, k­2∈Z) and there are some numbers which can satisfy the (k1m+k2n=1)

So, d∈Sx, y

Due to z is the smallest positive number of Sx, y.

So, z<=d.

Problem 3

1. (A\*B) \* (A\*B)

= (Ac∪Bc) \* (Ac∪Bc)

= (Ac∪Bc)c ∪ (Ac∪Bc)c

= (A∩B) ∪ (A∩B)

= A∩B

1. A \* A

A \* A

=Ac ∪ Ac

=Ac

1. (A\*(A\*A)) \* (A\*(A\*A))

(A\*(A\*A)) \* (A\*(A\*A))

=(A\*Ac) \* (A\*Ac)

=(A∩Ac)c \* (A∩Ac)c

=u \* u

=(u∩u)c

= Ø

1. (A\*(B\*B)) \* (A\*(B\*B))

(A\*(B\*B)) \* (A\*(B\*B))

=(A\*Bc) \* (A\*Bc)

=(A∩Bc)c \* (A∩Bc)c

=(A∩Bc) ∪ (A∩Bc)

= A∩Bc

=A\B

Problem4

1. w=a, v=b.

when w=a, v=b, there is no such z ∈ Σ\* that can satisfy v=wz.

1. {λ, a, ab, aba}

For relation R, there are {(λ, aba), (a, aba), (ab, aba), (aba, aba)}

So, v = {λ, a, ab, aba}

1. If R is partial order, then R is Reflexive, Antisymmetric and Transitive.

For Reflexive, when z=λ, v=w, and (w, w) ∈ R, so R is Reflexive.

For Antisymmetric, (w, v) ∈R and (v, w) ∈R, then, v=wz­1 (z1∈ Σ\*) and w = vz2 (z2∈ Σ\*), v = vz1z2 (z1, z2 ∈ Σ\*), so, z1 = z2 =λ, so, R is Antisymmetric.

For Transitive, (w, v) ∈R and (v, p) ∈R, then, v=wz­1 (z1∈ Σ\*) and p = vz2 (z2∈ Σ\*), then, p = wz1z2 (z1, z2 ∈ Σ\*), so, (w, p) ∈R, so, R is Transitive.

From above, R is partial order.

Problem5

Because gcd(x, y)=1

Known from problem2, there are m, n, which can make mx + ny=1 (m, n∈Z)

So, mxz + nyz = z (m, n, x, y, z∈Z)

Because x | xz, so, x | mxz (m, x, y, z∈Z)

Because x | yz, so, x | nyz (n, x, y, z∈Z)

So, mxz = k1x and nyz = k2x (m, n, x, y, z, k1, k2∈Z)

So, z = (k1+k2)x (x, z, k1, k2∈Z)

So, x | z.